§8. Stable Sheath and Pre-sheath Formation in Expanding Magnetic Field

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The stable sheath formation in expanding magnetic field to a divertor plate was theoretically studied using one-dimensional analysis. In fusion devices the magnetic field is expanding in the direction of the divertor plate, i.e. the magnitude of magnetic field is decreasing to the plate. The one dimensional analysis gives the the condition for the stable Debye sheath formation in expanding magnetic field to the mean ion velocity at the plasma-sheath boundary $\langle v_z \rangle_b$: [1)]

$$\langle v_z^{-2} \rangle_b^{-1} \geq \frac{ZT_e/M}{1-\alpha}, \tag{1}$$

the parameter α, which indicates the effect of a nonuniform magnetic field, is defined:

$$\alpha \equiv \frac{T_e}{e}\left(1+\frac{1}{2}\langle v_\perp^2/v_z^2 \rangle_b\right)\frac{d(\ln B)/dz}{d\phi/dz}\bigg|_b . \tag{2}$$

The subscript b indicates the value at the plasma-sheath boundary. In our case, where the strength of magnetic field is decreasing to the wall as well as electrostatic potential ϕ, the parameter α has a value of positive definite, which means the generalized Bohm criterion for the case of nonuniform magnetic field is restricted compared to that of the uniform magnetic field.

$$\langle v_z^{-2} \rangle_b^{-1} \geq \frac{ZT_e/M}{1-\alpha} \geq \frac{ZT_e}{M}. \tag{3}$$

The difference, however, between both cases is an order of the Debye length to a plasma radius, which is negligible small.

The requirement for the ion flow velocity inside the quasi-neutral plasma in order to form the stable pre-sheath is obtained from the condition of the quasi-neutrality.

$$\frac{ZT_e}{M}\langle v_z^{-2} \rangle_{in} = \frac{1}{1-\delta_s}\left[1+\delta_s\frac{ZT_e}{T_s} - (1-\delta_s)\frac{T_e}{e}\left(1+\frac{1}{2}\langle v_\perp^2/v_z^2 \rangle_{in}\right)\frac{d(\ln B)/dz}{d\phi/dz}\bigg|_{in}\right] \tag{4}$$

where δ_s is defined by the ratio of the density of the source particle to the electron density : $\delta_s \equiv Zn_{is}/n_e$. The values denoted by the subscript *in* is evaluated at the plasma injection point to the quasi-neutral plasma. The temperatures of electrons T_e and source particles T_s are assumed uniform inside the plasma region. The third term of RHS of eq.(4), which designates the non-uniformity of the magnetic field, is the order of unity because of the same scale length of the magnetic field and the electrostatic potential. On the other hand, the second term of RHS of eq.(4), which indicates the effects of the particle source, is much larger than unity because of higher plasma temperature compared to that of source particle. In case of the monoenegetic ion distribution of the injected ions:

$$f_{in} = n_{in0}\,\delta(v_\perp)\,\delta(v_z - v_{z0}) , \tag{5}$$

the relation between potential and magnetic field is obtained:

$$\frac{d(\ln B)}{-ed\phi/T_e} = \frac{(1-\delta_s)\frac{ZT_e}{M}v_{z0}^{-2} - 1 - \delta_s\frac{ZT_e}{T_s}}{(1-\delta_s)}, \tag{6}$$

which should be negative for the decreasing potential and magnetic field to the divertor plate, i.e.:

$$v_{z0}^2/\left(\frac{ZT_e}{M}\right) > \frac{1-\delta_s}{1+(Z\delta_s T_e/T_s)}. \tag{7}$$

Without plasma source inside quasi-neutral plasma ($\delta_s = 0$) , this relation leads the generalized Bohm's criterion:

$$v_{z0}^2/\left(\frac{ZT_e}{M}\right) > 1. \tag{8}$$

The effect of ion source is obtained in Fig. 1 for the case of $Z=1$, $T_e = 10\,eV$, and $T_s = 0.03eV$, which corresponds of room temperature of $20C^\circ$.

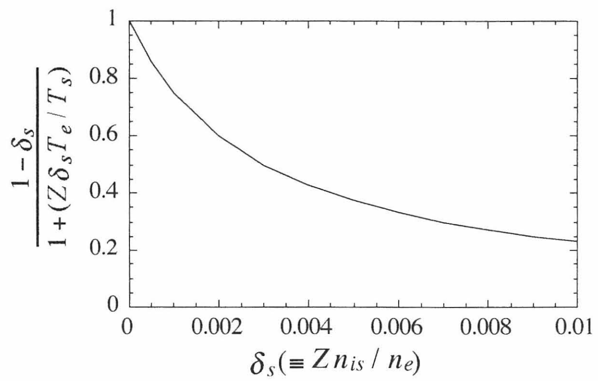

Fig.1. The limit of ion flow velocity compared with ion sound velocity, e.i. eq. (7), as a function of source particle concentration.

This result shows ion source inside the plasma mitigates the generalized Bohm's criterion.

References

1) Y. Tomita, A. Takayama, H. Takamaru, and T. Sato, J. Plasma Fusion Res. SERIES, 4 (2001) 578.